Effect of Thermal Fluctuation on Spectral Function for the Tomonaga-Luttinger Model*)

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We examine the spectral function of the single electron Green function at finite temperatures for the Tomonaga-Luttinger model which consists of the mutual interaction with only the forward scattering. The spectral weight, which is calculated as a function of the frequency with the fixed wave number, shows that several peaks originating in the excitation spectra of charge and spin fluctuations vary into a single peak by the increase of temperature.

§1. Introduction

The Tomonaga-Luttinger model [1),2)], which can be solved exactly, is a fundamental model for studying the electronic state in one-dimensional interacting electron systems. Since the excitations are gapless for both charge and spin degree of freedom, the response functions for the superconducting state, spin density wave state and charge density wave state show the power law behavior with respect to frequency and temperature where the exponents depend on the interactions.[3),4)]

Noticeable properties of the Tomonaga-Luttinger model have been obtained in the state of the single electron. The power law dependence has been found in the momentum distribution function[5)] around the Fermi momentum and the density of states[6),7),8)] around the Fermi energy. These facts are characteristic of the Luttinger liquid[9)] where the quantum fluctuation in one-dimension leads to the marginal behavior between the metallic state and the ordered state. The detail of the electronic state is obtained in the spectral function which is calculated from the imaginary part of the single electron Green function. The spectral function at absolute zero temperature exhibits several peaks which are associated with the fluctuation of the pairing electron and the separation of the charge and spin degrees of freedom in the presence of the interaction.[10),8)] The spectral function for the Hubbard model with the quarter-filled band and the infinite repulsive interaction[11)] has been calculated where the frequency-dependence is similar to that of the Luttinger liquid. However the temperature-dependence of the spectral function of the Tomonaga-Luttinger model is not studied qualitatively although the thermal fluctuation takes an important role for the electronic state around the Fermi energy.[6),7)]

In the present paper, we examine the spectral function at finite temperatures to understand how several peaks are varied by the thermal fluctuation. In §2, formulation is given in terms of the single electron Green function at finite temperatures. In §3, the frequency-dependence of the spectral function is examined numerically

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with some choices of interaction and momentum. The crossover from the ground state to the state expected at finite temperature is demonstrated. §4 is devoted to discussion.

§2. Formulation

The Tomonaga-Luttinger model is given by

$$\begin{aligned} H = & \sum_{r=\pm}\sum_{s=\uparrow,\downarrow}\sum_{k} v_{\rm F}(rk-k_{\rm F})C^{+}_{k,s,r}C_{k,s,r} \\ & +\frac{\pi v_{\rm F}}{2L}\sum_{r,s,s'}\sum_{k_1,k_2,q} {\rm e}^{-|q|\Lambda}\Bigg[\left(\widetilde{g}_{2\parallel}\delta_{s,s'}+\widetilde{g}_{2\perp}\delta_{s,-s'}\right)C^{+}_{k_1,s,r}C^{+}_{k_2,s',-r}C_{k_2+q,s',-r}C_{k_1-q,s,r} \\ & +\widetilde{g}_{4\perp}\delta_{s,-s'}\left[C^{+}_{k_1,s,r}C^{+}_{k_2,s',r}C_{k_2+q,s',r}C_{k_1-q,s,r}\right]\Bigg] \ , \end{aligned} \tag{2·1}$$

where $C^{+}_{k,s,r}$ denotes the creation operator of the fermion with spin $s=\uparrow(\downarrow)(=+(-))$ and the momentum k being positive (negative) for $r=+(-)$. The first term is the kinetic energy where $v_{\rm F}$ and $k_{\rm F}$ are Fermi velocity and Fermi momentum respectively. In the second term, the quantity $\widetilde{g}_{2\parallel,\perp}$ $(\equiv g_{2\parallel,\perp}/\pi v_{\rm F})$ denotes the normalized coupling constant of the interaction for the forward scattering between two kinds of electrons with $r=+$ and $r=-$ and $\widetilde{g}_{4\perp}$ $(\equiv g_{4\perp}/\pi v_{\rm F})$ is that for the same kind of electrons. Quantities Λ^{-1} and L denote the momentum cutoff of the interaction and the length of the system respectively.

Based on the bosonization method,[5] eq. (2·1) in case of $|q| \lesssim \Lambda^{-1}$ is expressed as[12]

$$H_P = \sum_{\nu=\rho,\sigma}\frac{v_\nu}{4\pi}\int_{-\infty}^{\infty}{\rm d}x\left[\frac{1}{\eta_\nu}\left(\partial_x\theta_{\nu,+}(x)\right)^2+\eta_\nu\left(\theta_{\nu,-}(x)\right)^2\right] \ , \tag{2·2}$$

$$v_\nu = v_{\rm F}\sqrt{(1\pm\widetilde{g}_{4\perp}/2)^2-\left((\widetilde{g}_{2\parallel}\pm\widetilde{g}_{2\perp})/2\right)^2} \ , \tag{2·3}$$

$$\eta_\nu = \sqrt{\frac{1\pm\widetilde{g}_{4\perp}/2-(\widetilde{g}_{2\parallel}\pm\widetilde{g}_{2\perp})/2}{1\pm\widetilde{g}_{4\perp}/2+(\widetilde{g}_{2\parallel}\pm\widetilde{g}_{2\perp})/2}} \ , \tag{2·4}$$

where $+(-)$ corresponds to $\rho(\sigma)$ and

$$\theta_{\rho,\pm}(x) = \frac{i}{2}\sum_q\frac{2\pi}{Lp}{\rm e}^{-\frac{\alpha_0}{2}|q|-iqx}\sum_{k,s}\left[C^{+}_{k+q,s,+}C_{k,s,+}\pm C^{+}_{k+q,s,-}C_{k,s,-}\right], \tag{2·5}$$

$$\theta_{\sigma,\pm}(x) = \frac{i}{2}\sum_q\frac{2\pi}{Lq}{\rm e}^{-\frac{\alpha_0}{2}|q|-iqx}\sum_{k,s}s\left[C^{+}_{k+q,s,+}C_{k,s,+}\pm C^{+}_{k+q,s,-}C_{k,s,-}\right]. \tag{2·6}$$

Equations (2·5) and (2·6), which denote the phase variables for the charge and spin fluctuations respectively, satisfy the commutation relation given by

$$[\theta_{\nu,\pm}(x),(-1/2\pi)\partial_{x'}\theta_{\nu',\mp}(x')] = i\delta_{\nu,\nu'}\delta(x-x') \ . \tag{2·7}$$

From eqs. (2·2) and (2·7), one obtains the excitation spectra, $v_\rho q$ and $v_\sigma q$ for the charge fluctuation and the spin fluctuation respectively. By introducing the cut-off parameter $\alpha_0(\to +0)$ for the convergence and making use of the fermion field operator defined as [3)]

$$\Psi^P_{s,r}(x,t) = \frac{\mathrm{e}^{rik_\mathrm{F}x}}{\sqrt{2\pi\alpha_0}}\exp\left[\frac{ri}{2}\left[(\theta_{\rho,+}(x,t)+r\theta_{\rho,-}(x,t))+s\left(\theta_{\sigma,+}(x,t)+r\theta_{\sigma,-}(x,t)\right)\right]\right], \tag{2·8}$$

the retarded Green function at finite temperatures is calculated as [6)]

$$\begin{aligned} G^R_{s,r}(x,t) &\equiv -i\theta(t)\left\langle \Psi^P_{s,r}(x,t)\Psi^P_{s,r}(0,0)^+ + \Psi^P_{s,r}(0,0)^+\Psi^P_{s,r}(x,t)\right\rangle_{H_P} \\ &= -i\frac{\theta(t)}{2\pi}e^{irk_\mathrm{F}x}\left[\prod_{\nu=\rho,\sigma}\left[\frac{1}{(\alpha_0+i(v_\nu t-rx))^{\frac{1}{2}}}\left(\frac{\Lambda^2}{(\Lambda+iv_\nu t)^2+x^2}\right)^{\gamma_\nu}\right]\right. \\ &\qquad \left.\times\Xi(x,t,T)+\begin{pmatrix} x\to -x \\ t\to -t\end{pmatrix}\right] \quad , \end{aligned} \tag{2·9}$$

$$\begin{aligned} &\Xi(x,t,T) \\ &= \prod_{\nu=\rho,\sigma}\prod_{n=1}^{\infty}\left[\left[1+\left(\frac{v_\nu t-rx}{\alpha_0+\frac{nv_\nu}{T}}\right)^2\right]^{\frac{1}{2}}\left[1+\left(\frac{v_\nu t-rx}{\Lambda+\frac{nv_\nu}{T}}\right)^2\right]^{\gamma_\nu}\left[1+\left(\frac{v_\nu t+rx}{\Lambda+\frac{nv_\nu}{T}}\right)^2\right]^{\gamma_\nu}\right]^{-1} . \end{aligned} \tag{2·10}$$

The quantity γ_ν, which denotes the magnitude of the interaction, is defined by

$$\gamma_\nu = (\eta_\nu + \eta_\nu^{-1} - 2)/8 \ . \tag{2·11}$$

The quantity, T, is the temperature and k_B is taken as unity. In deriving eq. (2·9) from eq. (2·8), we have used an approximation that the second line of eq. (2·9) is correct in case of $|x|, |v_\mathrm{F}t| \gg \Lambda$ and results in the extra factor of $v_\mathrm{F}/(v_\rho v_\sigma)^{1/2}$ in case of $|x|, |v_\mathrm{F}t| \ll \Lambda$ compared with the exact one. [13), 6)] Actually, by noting that $\Xi(x,t,T) \to 1$ in the limit of absolute zero temperature, eq. (2·9) in case of $T = 0$ becomes equal to the retarded Green function obtained by Luther and Peschel [3), 10), 8)] which is valid for the length scale being larger than Λ. Therefore the following calculation of the spectral function is justified when the frequency (the momentum) is smaller than v_F/Λ (Λ^{-1}). We note the difference between two kinds of cutoff parameters in the Tomonaga-Luttinger model. The quantity Λ leading to the momentum cutoff of the interaction plays an essential role for the existence of the characteristic energy, v_F/Λ, but the quantity α_0 vanishes in the end by taking the limit of $\alpha_0 \to 0$.

In terms of eq. (2·9), the spectral function is calculated as

$$\begin{aligned} A_r(q,\omega) &= -\frac{1}{\pi}\mathrm{Im}\left[\int_{-\infty}^{\infty}\mathrm{d}x\int_{-\infty}^{\infty}\mathrm{d}t\mathrm{e}^{-i\{(q+rk_\mathrm{F})x-\omega t\}}G^R_r(x,t)\right] \\ &= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\mathrm{d}x\int_{-\infty}^{\infty}\mathrm{d}t\Big[\mathrm{e}^{-i(qx-\omega t)} \end{aligned}$$

$$\times \prod_{\nu=\rho,\sigma}\left[\frac{1}{(\alpha_0+i(v_\nu t-rx))^{\frac{1}{2}}}\left(\frac{\Lambda^2}{(\Lambda+iv_\nu t)^2+x^2}\right)^{\gamma_\nu}\right]\Xi(x,t,T)$$
$$+\begin{pmatrix}\omega\to-\omega\\ q\to-q\end{pmatrix}\Bigg] \,. \tag{2·12}$$

We examine eq. (2·12) for two kinds of cases. The case (i) is given by $\widetilde{g}_{2\parallel}\equiv\widetilde{g}\neq 0$ and $\widetilde{g}_{2\perp}=\widetilde{g}_{4\perp}=0$ which corresponds to the spinless Tomonaga-Luttinger model. The case (ii) is given by $\widetilde{g}_{2\parallel}=\widetilde{g}_{2\perp}=\widetilde{g}_{4\perp}\equiv\widetilde{g}\neq 0$ which represents the spinful Tomonaga-Luttinger model.

In the case (i), eq. (2·12) is rewritten as

$$A_r(q,\omega)=\frac{1}{8\pi^2 v}\left[F_1(\Omega_1,T)\,F_2(\Omega_2,T)+\begin{pmatrix}\Omega_1\to-\Omega_1\\ \Omega_2\to-\Omega_2\end{pmatrix}\right] \,. \tag{2·13}$$

By use of $\Omega_1=(\omega+rvq)/2v$, $\Omega_2=(\omega-rvq)/2v$, $s_1=vt-rx$ and $s_2=vt+rx$, quantities $F_1(\Omega_1,T)$ and $F_2(\Omega_2,T)$ are written as

$$\begin{aligned}
&F_1(\Omega_1,T)\\
&=\int_{-\infty}^{\infty}\mathrm{d}s_1\frac{\mathrm{e}^{i\Omega_1 s_1}}{\alpha_0+is_1}\left(\frac{\Lambda}{\Lambda+is_1}\right)^{\gamma}\times\prod_{n=1}^{\infty}\left[\left[1+\left(\frac{s_1}{\alpha_0+\frac{nv}{T}}\right)^2\right]\left[1+\left(\frac{s_1}{\Lambda+\frac{nv}{T}}\right)^2\right]^{\gamma}\right]^{-1}\\
&=\pi+2\Lambda^{\gamma}\int_0^{\infty}\mathrm{d}s_1\frac{\sin\left[\Omega_1 s_1-\gamma\tan^{-1}\left(\frac{s_1}{\Lambda}\right)\right]}{s_1\left(\Lambda^2+s_1^2\right)^{\frac{\gamma}{2}}}\frac{Ts_1/v}{\sinh\left(Ts_1/v\right)}\prod_{n=1}^{\infty}\left[1+\left(\frac{s_1}{\Lambda+\frac{nv}{T}}\right)^2\right]^{-\gamma},
\end{aligned} \tag{2·14}$$

$$\begin{aligned}
&F_2(\Omega_2,T)\\
&=\int_{-\infty}^{\infty}\mathrm{d}s_2\mathrm{e}^{i\Omega_2 s_2}\left(\frac{\Lambda}{\Lambda+is_2}\right)^{\gamma}\prod_{n=1}^{\infty}\left[1+\left(\frac{s_2}{\Lambda+\frac{nv}{T}}\right)^2\right]^{-\gamma}\\
&=2\Lambda^{\gamma}\int_0^{\infty}\mathrm{d}s_2\frac{\cos\left[\Omega_2 s_2-\gamma\tan^{-1}\left(\frac{s_2}{\Lambda}\right)\right]}{\left(\Lambda^2+s_2^2\right)^{\frac{\gamma}{2}}}\prod_{n=1}^{\infty}\left[1+\left(\frac{s_2}{\Lambda+n\frac{v}{T}}\right)^2\right]^{-\gamma},
\end{aligned} \tag{2·15}$$

where $v=v_\rho=v_\sigma=v_\mathrm{F}[1-(\widetilde{g}_2/2)^2]^{1/2}$ and $\gamma=[(1-(\widetilde{g}_2/2)^2)^{-1/2}-1]/2$.

In the case (ii), eq. (2·12) is rewritten as

$$\begin{aligned}
A_r(q,\omega)&=\frac{1}{(2\pi)^2|v_\rho-v_\sigma|}\int_{-\infty}^{\infty}\mathrm{d}s_\rho\int_{-\infty}^{\infty}\mathrm{d}s_\sigma\Bigg[\mathrm{e}^{i(\Omega_\sigma s_\rho+\Omega_\rho s_\sigma)}\\
&\quad\times\prod_{\nu=\rho,\sigma}\left[\frac{1}{(\alpha_0+is_\nu)^{\frac{1}{2}}}\left(\frac{\Lambda}{\Lambda+is_\nu}\right)^{\gamma_\nu}\left(\frac{\Lambda}{\Lambda+i(a_\nu s_\rho+b_\nu s_\sigma)}\right)^{\gamma_\nu}\right]\Xi(x,t,T)\\
&\quad+\begin{pmatrix}\Omega_\rho\to-\Omega_\rho\\ \Omega_\sigma\to-\Omega_\sigma\end{pmatrix}\Bigg]\\
&=\frac{\Lambda^{2(\gamma_\rho+\gamma_\sigma)}}{(2\pi)^2|v_\rho-v_\sigma|}\left[\left[F_s(\Omega_\rho,\Omega_\sigma,T)+F_c(\Omega_\rho,\Omega_\sigma,T)\right]+\begin{pmatrix}\Omega_\rho\to-\Omega_\rho\\ \Omega_\sigma\to-\Omega_\sigma\end{pmatrix}\right] \,.
\end{aligned} \tag{2·16}$$

By use of $\Omega_\sigma = (\omega - rv_\sigma q)/(v_\rho - v_\sigma)$, $\Omega_\rho = (rv_\rho q - \omega)/(v_\rho - v_\sigma)$, $s_\rho = v_\rho t - rx$, $s_\sigma = v_\sigma t - rx$, $a_\rho = (v_\rho + v_\sigma)/(v_\rho - v_\sigma)$, $b_\rho = -2v_\rho/(v_\rho - v_\sigma)$, $a_\sigma = 2v_\sigma/(v_\rho - v_\sigma)$, $b_\sigma = -(v_\rho + v_\sigma)/(v_\rho - v_\sigma)$, quantities $F_s(\Omega_\rho, \Omega_\sigma, T)$ and $F_c(\Omega_\rho, \Omega_\sigma, T)$ are expressed as

$$F_s(\Omega_\rho, \Omega_\sigma, T) = 2\int_0^\infty \mathrm{d}s_\rho \int_0^\infty \mathrm{d}s_\sigma \Big[\prod_{\nu=\rho,\sigma} s_\nu (\Lambda^2 + s_\nu^2)^{\gamma_\nu} (\Lambda^2 + (a_\nu s_\rho + b_\nu s_\sigma)^2)^{\gamma_\nu} \Big]^{-\frac{1}{2}}$$
$$\times \sin\left[\Omega_\sigma s_\rho + \Omega_\rho s_\sigma - \sum_{\nu=\rho,\sigma} \gamma_\nu \left[\tan^{-1}\left(\frac{s_\nu}{\Lambda}\right) + \tan^{-1}\left(\frac{a_\nu s_\rho + b_\nu s_\sigma}{\Lambda}\right) \right] \right]$$
$$\times \xi(s_\rho, s_\sigma, T) \ , \tag{2·17}$$

$$F_c(\Omega_\rho, \Omega_\sigma, T) = 2\int_0^\infty \mathrm{d}s_\rho \int_{-\infty}^0 \mathrm{d}s_\sigma \Big[\prod_{\nu=\rho,\sigma} |s_\nu| (\Lambda^2 + s_\nu^2)^{\gamma_\nu} (\Lambda^2 + (a_\nu s_\rho + b_\nu s_\sigma)^2)^{\gamma_\nu} \Big]^{-\frac{1}{2}}$$
$$\times \cos\left[\Omega_\sigma s_\rho + \Omega_\rho s_\sigma - \sum_{\nu=\rho,\sigma} \gamma_\nu \left[\tan^{-1}\left(\frac{s_\nu}{\Lambda}\right) + \tan^{-1}\left(\frac{a_\nu s_\rho + b_\nu s_\sigma}{\Lambda}\right) \right] \right]$$
$$\times \xi(s_\rho, s_\sigma, T) \ , \tag{2·18}$$

where $\gamma_\rho = [(1 - (2\tilde{g}/(2+\tilde{g}))^2)^{-1/2} - 1]/4$, $\gamma_\sigma = 0$, $v_\rho = v_{\mathrm{F}}[(1+\tilde{g}/2)^2 - \tilde{g}^2]^{1/2}$ and $v_\sigma = v_{\mathrm{F}}(1 - \tilde{g}/2)$. In deriving eqs. (2·17) and (2·18), we made use of the following approximation,[14), 7)]

$$\Xi(x,t,T) \simeq \prod_{\nu=\rho,\sigma} \left[\left[\frac{T s_\mu / v_\nu}{\sinh(T s_\nu / v_\nu)} \right]^{\frac{1}{2}+\gamma_\nu} \left[\frac{T(a_\nu s_\rho + b_\nu s_\sigma)/v_\nu}{\sinh(T(a_\nu s_\rho + b_\nu s_\sigma)/v_\nu)} \right]^{\gamma_\nu} \right]$$
$$\equiv \xi(s_\rho, s_\sigma, T) \ , \tag{2·19}$$

which was obtained by discarding Λ in eq. (2·10). Such a treatment is valid for $T \lesssim v_{\mathrm{F}}/\Lambda$ in the present calculation.

In addition to the case (ii), we examine the case of $\tilde{g}_{4\perp} \neq 0$ and zero otherwise which corresponds to the one-branch Luttinger liquid,[8)] i.e., the forward scattering within the same kind of electrons. In this case, the spectral function $A_+(q,\omega)$ is obtained by putting $\gamma_\rho = \gamma_\sigma = 0$ in eq. (2·16) where $A_+(q,\omega)$ also shows the separation of the charge and spin degrees of freedom.

§3. Spectral Function

We evaluate the spectral weight of eq.(2·12) which is normalized as,

$$\tilde{A}_+(\tilde{q}, \tilde{\omega}) = A_+(q,\omega) v_{\mathrm{F}} \Lambda^{-1} \ . \tag{3·1}$$

Quantities q, ω, v and T are also normalized as $\tilde{q} = q\Lambda$, $\tilde{\omega} = \omega/(v_{\mathrm{F}}\Lambda^{-1})$, $\tilde{v} = v/v_{\mathrm{F}}$ and $\tilde{T} = T/(v_{\mathrm{F}}\Lambda^{-1})$ respectively. From eq.(2·11), the parameter for the interaction is defined as

$$\alpha = 2\,(\gamma_\rho + \gamma_\sigma) \ . \tag{3·2}$$

We examine $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ in detail by choosing α= 0.125 which corresponds to the limit of the large repulsive interaction for one-dimensional Hubbard model.[15] Since $A_r(\widetilde{q},\widetilde{\omega}) = A_r(-\widetilde{q},-\widetilde{\omega}) = A_{-r}(\widetilde{q},-\widetilde{\omega})$, we investigate numerically $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ in case of $\widetilde{q} > 0$.

3.1. *spinless case*

The model in the case of $g_{2\parallel} \neq 0$ and $g_{2\perp} = g_{4\perp} = 0$ is equivalent to that of the spinless fermion since there is no distinction between the charge fluctuation and the spin fluctuation, i.e., $\gamma_\rho = \gamma_\sigma$.

First, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with $\widetilde{q} = 0$ is examined.

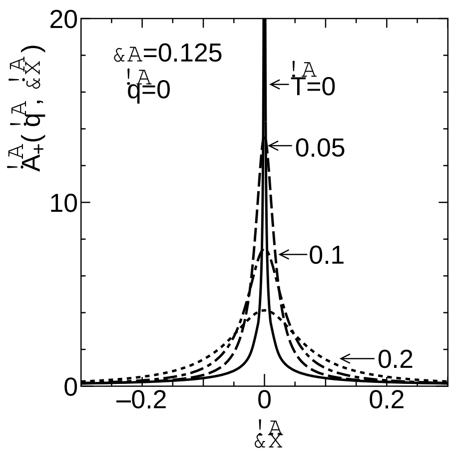

Fig. 1. The $\widetilde{\omega}$-dependence of spectral function $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ of the spinless model where $\widetilde{q} = 0$, $\alpha = 0.125$ and $\widetilde{T}$ is chosen as $\widetilde{T}$=0, 0.05, 0.1 and 0.2.

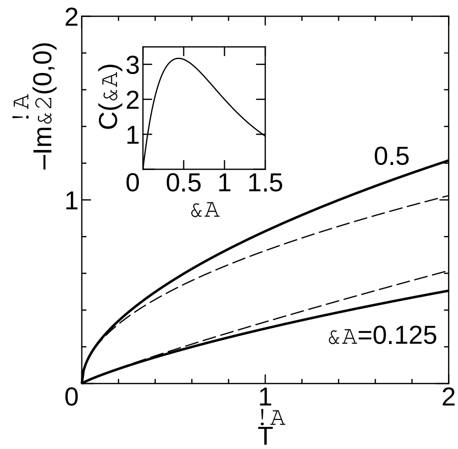

Fig. 2. The $\widetilde{T}$-dependence of the imaginary part of the normalized self-energy, $-\mathrm{Im}\widetilde{\Sigma}^R(0,0)$ $(= -\mathrm{Im}\Sigma^R(0,0)\Lambda/v_{\mathrm{F}})$ is shown in the case of $\alpha = 0.125$ and 1.5. The dashed curve is the asymptotic value given by eq. (3·3) and the inset shows the coefficient given by eq. (3·4).

In Fig. 1, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with $\alpha = 0.125$ and $\widetilde{q} = 0$ is shown as a function of $\widetilde{\omega}$ with some choices of $\widetilde{T}$. In case of $T = 0$, the spectral function $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ diverges at $\widetilde{\omega} = 0$ where $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with $\widetilde{q} = 0$ and small $|\widetilde{\omega}|$ is given by $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) \propto \widetilde{\omega}^{\alpha-1}$.[10),8)] For $\widetilde{T} \neq 0$, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with $\widetilde{\omega} = 0$ becomes finite and the peak of $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ decreases and the width increases due to the thermal fluctuation. The half width of $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$, which is defined by $\Delta\widetilde{\omega}$, is proportional to $\widetilde{T}$ within the numerical accuracy of the present calculation, e.g. $\Delta\widetilde{\omega} \sim 0.4\widetilde{T}$ $(\sim 2\widetilde{T}$) for $\widetilde{T} < 0.1$ in case of $\alpha = 0.125(= 0.5)$. The height of the peak at low temperatures is given by $\widetilde{A}_+(0,0) \propto \widetilde{T}^{\alpha-1}$ which is related to the imaginary part of the self-energy of the Fourier transform of the retarded Green function, eq. (2·9). Actually, by defining $\mathrm{Im}\Sigma^R(q,\omega)$ as the imaginary part of the Green function, one finds the relation that $\mathrm{Im}\Sigma^R(0,0) = -[\pi A(0,0)]^{-1}$ where $\mathrm{Re}\Sigma^R(0,0) = 0$. From eq. (2·13), the quantity $\mathrm{Im}\Sigma^R(0,0)$ in case of $T \ll v\Lambda^{-1}$ is

calculated as

$$-\,\mathrm{Im}\Sigma^R(0,0) \simeq C(\alpha)\left(\frac{T\Lambda}{v}\right)^{1-\alpha} \quad , \tag{3·3}$$

$$\begin{aligned} C(\alpha)^{-1} \quad &= \quad \frac{\pi^{\gamma-1}\sin(\pi\gamma)}{2v}\int_0^\infty \mathrm{d}y \left[\left(\frac{\pi}{\sinh(\pi y)}\right)^{\gamma+1} - y^{-(\gamma+1)}\right] \\ &\quad \times \int_0^\infty \mathrm{d}y\,(\sinh(\pi y))^{-\gamma} \quad . \end{aligned} \tag{3·4}$$

In Fig. 2, the normalized quantity of $\mathrm{Im}\Sigma^R(0,0)$ as a function of $\widetilde{T}$ is shown by the solid curves for $\alpha = 0.125$ and 0.5. The dashed curve denotes the asymptotic value given by eq. (3·3) where the good coincidence between the solid curve and the dashed curve is obtained at low temperatures. The actual value of $\mathrm{Im}\Sigma^R(0,0)$ is smaller (larger) than the asymptotic one value where the crossover takes place around $\alpha \simeq 0.32$ at low temperatures. In the inset, $C(\alpha)$ of eq. (3·4) is shown as a function of α. The quantity $C(\alpha)$ takes a maximum around $\alpha \simeq 0.44$ and $C(\alpha)$ becomes zero at $\alpha = 0$, indicating the fact that $A_r(q,\omega) \to \delta(\omega - v_{\mathrm{F}}q)$ at $\alpha \to 0$.

Next $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ in case of $\widetilde{q} = 0.1$ is examined. The $\widetilde{\omega}$-dependence of $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ in case of $\alpha = 0.125$ is shown in Fig. 3. The quantity $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ at $\widetilde{T} = 0$ shows not only the main peak around $\widetilde{\omega} = \widetilde{v}\widetilde{q}$ but also that near $\widetilde{\omega} = -\widetilde{v}\widetilde{q}$ where $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) \propto (\widetilde{\omega} - \widetilde{v}\widetilde{q})^{-1+\alpha/2}$ for $\widetilde{\omega} \gtrsim \widetilde{v}\widetilde{q}$, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) \propto |\widetilde{\omega} + \widetilde{v}\widetilde{q}|^{\alpha/2}$ for $\widetilde{\omega} \lesssim -\widetilde{v}\widetilde{q}$ and $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) = 0$ for $|\widetilde{\omega}| < \widetilde{v}\widetilde{q}$.[10),8)] The peak around $\widetilde{\omega} = -\widetilde{v}\widetilde{q}$ comes from the particle-hole excitations between two kinds of electrons with $r = \pm$.[8)] The finite magnitude of $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ appears in the interval region of $|\widetilde{\omega}| < \widetilde{v}\widetilde{q}$ at finite temperatures. By the increase of $\widetilde{T}$, these two peaks are suppressed and merges into a single peak.

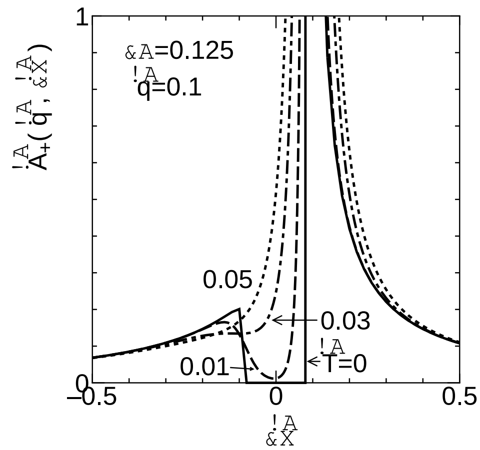

Fig. 3. The $\widetilde{\omega}$-dependence of spectral function $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ of the spinless model where $\widetilde{q} =$ 0.1, $\alpha = 0.125$ and $\widetilde{T}$ is chosen as $\widetilde{T}$=0, 0.01, 0.02, 0.05 and 0.1.

3.2. *spinful case*

We examine $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ in the presence of the interactions $g_{2\parallel}$, $g_{2\perp}$ and $g_{4\perp}$ which result in the separation of the charge degree of freedom from the spin degree of freedom. When $\widetilde{q} = 0$, the $\widetilde{\omega}$-dependence of $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ is similar to Fig. 1 since the excitation spectra of both charge fluctuation and spin fluctuation become equal to zero at $\widetilde{q} = 0$. In Fig. 4, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ in case of $\widetilde{q} = 0.5$ and α=0.125 is shown with some choices of $\widetilde{T}$ where $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with $\widetilde{\omega} < 0$ is multiplied by 10. In case of $\widetilde{T} = 0$, there are several kinds of edges in $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ which originate in excitation spectra of the spin and charge fluctuations. Their asymptotic forms in the case of $\gamma_\sigma = 0$ are

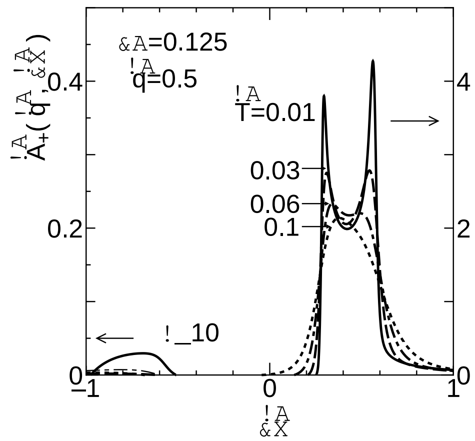

Fig. 4. The $\widetilde{\omega}$-dependence of spectral function $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ of the spinful model where $\widetilde{q} = 0.5$, $\alpha = 0.125$ and $\widetilde{T}$ is chosen as $\widetilde{T}=$ 0.01(solid curve), 0.03(dashed curve), 0.06(dash-dotted curve) and 0.1(dotted curve).

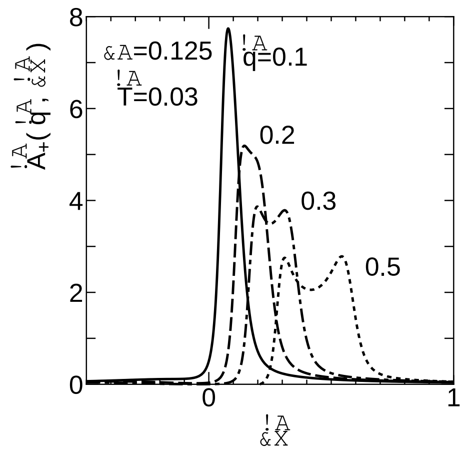

Fig. 5. The $\widetilde{\omega}$-dependence of spectral function $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ of the spinlful model where $\alpha =$ 0.125, $\widetilde{T} = 0.03$ and $\widetilde{q}$ is chosen as $\widetilde{q}$=0.1, 0.2, 0.3 and 0.5.

given by [10), 8)]

$$A_+(q,\omega)|_{\omega\simeq v_\rho q} \propto |\omega - v_\rho q|^{\gamma_\rho+2\gamma_\sigma-1/2} \quad ,$$
$$A_+(q,\omega)|_{\omega\simeq v_\sigma q} \propto \theta(\omega - v_\sigma q)(\omega - v_\sigma q)^{2\gamma_\rho+\gamma_\sigma-1/2} \quad ,$$
$$A_+(q,\omega)|_{\omega\simeq -v_\rho q} \propto \theta(-\omega - v_\rho q)|\omega + v_\rho q|^{\gamma_\rho+2\gamma_\sigma} \quad , \qquad (3\cdot5)$$

and $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) = 0$ for $-v_\rho\widetilde{q} < \widetilde{\omega} < v_\sigma\widetilde{q}$.

The $\widetilde{T}$-dependence of $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ is examined by defining $\widetilde{A}_{+,\rho}$ ($\widetilde{A}_{+,\sigma}$) as $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ corresponding to the peak located near $\omega = v_\rho q$ ($\omega = v_\sigma q$) where $v_\sigma q < v_\rho q$. The result that $\widetilde{A}_{+,\sigma} < \widetilde{A}_{+,\rho}$ in case of $\widetilde{T} = 0.01$ can be understood from the fact that, at $\widetilde{T} = 0$, the exponent for the divergence of the charge excitation is larger than that of the spin excitation as is seen from eqs. (3·5). By the increase of $\widetilde{T}$, one finds that $\widetilde{A}_{+,\sigma} \simeq \widetilde{A}_{+,\rho}$ at $\widetilde{T}$=0.03 and that $\widetilde{A}_{+,\sigma} > \widetilde{A}_{+,\rho}$ for $\widetilde{T}$=0.06. Such a crossover from the dominant $\widetilde{A}_{+,\rho}$ into the dominant $\widetilde{A}_{+,\sigma}$ is characteristic of the finite temperature. The spin excitation, which has the energy lower than that of the charge excitation, has the large effect on $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ and give rise to the dominant $\widetilde{A}_{+,\rho}$ as is seen from the interference term, $\Omega_\rho s_\sigma + \Omega_\sigma s_\rho$, in eq. (2·16). The quantity $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ at $\widetilde{T} = 0.1$ shows that the two peaks at low temperatures becomes a single peak with the broad width. When $\widetilde{T}$ increases, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with $\omega < 0$ decreases indicating the fact that the correlation by the interaction decreases by the thermal fluctuation. In Fig. 5, we show $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with the fixed $\widetilde{T}$ by choosing several $\widetilde{q}$. The interval length of $\widetilde{\omega}$ between two peaks, which exists for the large $\widetilde{q}$, decreases by the decrease of $\widetilde{q}$ and vanishes for the small $\widetilde{q}$, e.g., $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ for $\widetilde{\omega} = 0.2$ and 0.1.

For the comparison, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with the interaction of only $g_{4\perp}$ is shown in Fig. 6. In case of $T = 0$, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ exists only in the interval region of $v_\sigma q < \omega < v_\rho q$ and show two singularities at the boundaries corresponding to the spin and charge excitations respectively. The exponents for the divergence at the charge excitation is the same as the spin excitation where $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) \propto (\widetilde{\omega} - \widetilde{v}_\sigma\widetilde{q})^{-1/2}$ for $\widetilde{\omega} \gtrsim \widetilde{v}_\sigma\widetilde{q}$, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) \propto (-\widetilde{\omega} + \widetilde{v}_\rho\widetilde{q})^{-1/2}$ for $\widetilde{\omega} \lesssim \widetilde{v}_\rho\widetilde{q}$ and $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) = 0$ for $\widetilde{\omega} < \widetilde{v}_\sigma\widetilde{q}$ and $\widetilde{\omega} > \widetilde{v}_\rho\widetilde{q}$.[8)] The effect of the thermal fluctuation, which leads to the crossover from two peaks to the single peak, is similar to Fig. 5. When $\omega = v_\sigma q$ or $\omega = v_\sigma q$, eq. (2·12) is rewritten as

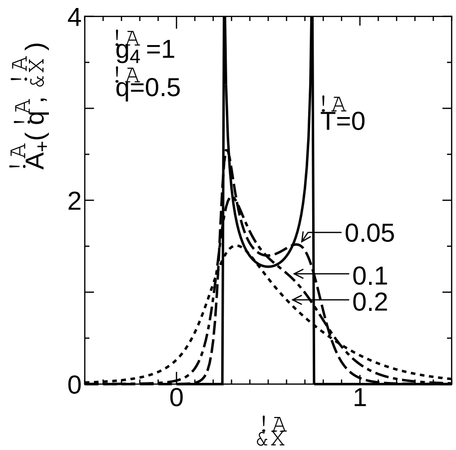

Fig. 6. The $\widetilde{\omega}$-dependence of spectral function $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with the interaction, $g_{4\perp}$, where $\widetilde{q} = 0.5$, $\widetilde{g}_4 = g_{4\perp}/(\pi v_{\rm F})$ and $\widetilde{T}$ is chosen as $\widetilde{T}$=0, 0.05, 0.1 and 0.2.

$$A_+(q, v_\sigma q) = \frac{C_1}{T}\int_0^\infty {\rm d}y \frac{\cos\left(\frac{v_\sigma q}{T}y\right)}{(\sinh y)^{\frac{1}{2}}} \ , \tag{3·6}$$

$$A_+(q, v_\rho q) = \frac{C_1}{T}\int_0^\infty {\rm d}y \frac{\cos\left(\frac{v_\rho q}{T}y\right)}{(\sinh y)^{\frac{1}{2}}} \ , \tag{3·7}$$

where $C_1 = \sqrt{v_\rho v_\sigma}\pi^{-2}|v_\rho - v_\sigma|^{-1}\int_0^\infty {\rm d}y\,(\sinh y)^{-\frac{1}{2}}$. Since eqs. (3·6) $\propto T^{-1/2}v_\sigma^{-1}$ and eqs. (3·7) $\propto T^{-1/2}v_\rho^{-1}$ at low temperatures, it turns out that $\widetilde{A}_{+,\rho}$ decreases rapidly compared with $\widetilde{A}_{+,\sigma}$.

§4. Discussion

We examined the spectral function, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$, of the Tomonaga-Luttinger model at finite temperatures for both the spinless case and the spinful case by choosing $\alpha = 0.125$ which corresponds to the large limit of the repulsive interaction of the Hubbard model. In case of $\widetilde{q} = 0$, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ shows the peak around $\widetilde{\omega} = 0$. By the increase of temperature, the height decreases and the width increases due to the thermal fluctuation. At low temperatures, $\widetilde{A}_+(\widetilde{q},\widetilde{\omega})$ with $\widetilde{q} = 0$ and $\widetilde{\omega} = 0$, shows the power law expressed as $\widetilde{A}_+(\widetilde{q},\widetilde{\omega}) \propto \widetilde{T}^{\alpha-1}$. In case of $\widetilde{q} > 0$, there are two peaks located in the region of $\omega > 0$ and that of $\omega < 0$ for both the spinless and spinful cases. With increasing temperature, these two peaks moves to a single peak around $\omega = vq$. In the spinful case, the peak with $\widetilde{q} > 0$ is separated into two peaks where the peak with the large $\widetilde{\omega}$, $\widetilde{A}_{+,\rho}$, corresponds to the charge fluctuation and the peak with the small $\widetilde{\omega}$, $\widetilde{A}_{+,\sigma}$, corresponds to the spin fluctuation. In the limit of low temperatures, one obtains $\widetilde{A}_{+,\sigma} < \widetilde{A}_{+,\rho}$, while one finds $\widetilde{A}_{+,\sigma} > \widetilde{A}_{+,\sigma}$ by the increase of temperature. The latter result, which is explained by eq. (2·16), can be also

understood in terms of the general formula [16]

$$A(k,\omega) = \frac{1}{Z}\sum_{m,n}\left[|(C_k^+)_{n,m}|^2 \mathrm{e}^{-\frac{1}{T}(E_m-\mu N_m)}(1+\mathrm{e}^{-\frac{\omega}{T}})\delta(E_n-E_m-\mu-\omega)\right], \quad (4\cdot 1)$$

where $Z=\sum_n \mathrm{e}^{(E_n-\mu N_n)/T}$ and N_n is the electron number. Quantities E_n and μ are the energy of the n-th eigenstate and the chemical potential respectively and $(C_k^+)_{n,m}$ denotes the matrix element between the n-th eigenstate and m-th eigenstate. Since $v_\rho q > v_\sigma q$ in the present case, the factor, $(1+\mathrm{e}^{-\omega/T})$, with $\omega = v_\sigma q$ is larger than the factor with $\omega = v_\rho q$. Therefore the peak for the spin fluctuation becomes larger than that for the charge fluctuation. At higher temperatures, these two peaks also merge into a single peak.

Finally, we comment on the experiment on $K_{0.3}MoO_3$ where the angle-resolved photoemission spectroscopy reveals the two peaks corresponding to the charge and spin separations. [17] As for two peaks indicating the properties of the Tomonaga-Luttinger liquid, the peak with the lower energy is larger than the peak with the higher energy. In the Tomonaga-Luttinger model, the peak with lower energy is rather suppressed when the interaction for the charge density is strong enough. Then Voit [18] claimed that the peak with the higher energy is rather suppressed by the backward scattering. [19] Here we comment another possibility that the suppression of the peak with higher energy is attributable to the effect of thermal fluctuation as is found in Figs. 4 and 6.

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